

Spectator Effects during Leptogenesis in the Strong Washout Regime

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Abstract

By including spectator fields into the Boltzmann equations for Leptogenesis, we show that partially equilibrated spectator interactions can have a significant impact on the freeze-out value of the asymmetry in the strong washout regime. The final asymmetry is typically increased, since partially equilibrated spectators "hide" a part of the asymmetry from washout. We study examples with leptonic and non-leptonic spectator processes, assuming thermal initial conditions, and find up to 50% enhanced asymmetries compared to the limit of fully equilibrated spectators. Together with a comprehensive overview of the equilibration temperatures for various Standard Model processes, the numerical results indicate the ranges when the limiting cases of either fully equilibrated or negligible spectator fields are applicable and when they are not. Our findings also indicate an increased sensitivity to initial conditions and finite density corrections even in the strong washout regime.

1 Introduction

In standard Leptogenesis, the asymmetry is created from out-of-equilibrium decays of heavy singlet neutrinos into lepton and Higgs doublets [1]. The Standard Model (SM) encompasses a whole cascade of interactions that equilibrate in the Early Universe subsequently at temperatures below $\sim 10^{15}$ GeV. Many of these processes can redistribute the initial asymmetries to spectator degrees of freedom, see *e.g.* Refs. [2, 3] for an overview. The spectator particles do not directly couple to the singlet neutrinos and are therefore not directly washed out. This can have a sizeable impact on the freeze-out value of the lepton asymmetry [2–6].

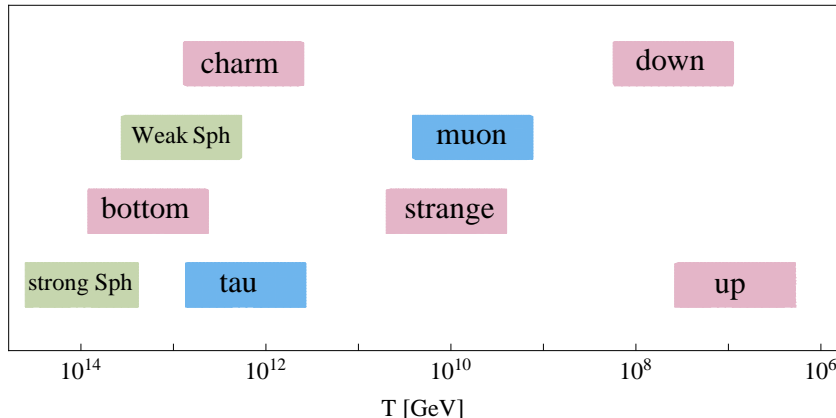


Figure 1: Temperature ranges where different SM interactions equilibrate. Shown are the strong and weak sphalerons (green), and quark (red) and lepton (blue) Yukawa interactions. The bands range from T_X to $20 T_X$, with T_X denoting the equilibration temperatures for the individual interactions that are calculated in more detail in Section 3. The bands indicate the range where one can expect nontrivial effects due to partial equilibration, based on the results of Sections 4 and 5.

As more spectator fields reach chemical equilibrium, the washout rate is more reduced, leading to an increase in the efficiency of the production of the asymmetry [2, 3]. The rates of the numerous SM interactions that mediate chemical equilibrium are spread over a large temperature range, which we illustrate in Figure 1. One should therefore expect that situations when spectator fields partially equilibrate (as opposed to the limiting cases when they are either negligible or fully equilibrated) are not exceptional. Quantifying the limitations of the approximations of negligible or fully equilibrated spectator effects and the proper treatment of the intermediate regime is the main purpose of the present work.

When shifting the freeze-out temperature to lower values, one may expect that the efficiency of Leptogenesis simply increases monotonously, because the spectator fields move more closely to chemical equilibrium. While for some regions of parameter space, in particular when washout is not too strong, this is indeed the case, there is however an interesting curiosity (that can be observed for the partial equilibration of τ -Yukawa interactions in the numerical analysis of Ref. [7]) that generically occurs for stronger washout: The freeze-out asymmetry for partially equilibrated spectator fields can be enhanced compared to the two limiting cases. This is because in the strong washout regime, large asymmetries are present at early times, that are then partially transferred to the spectator sector, where these are “hidden” from washout. At later times, in particular when washout processes freeze out, these asymmetries can exceed the asymmetries present in the case of full equilibration (where the spectators closely track the asymmetry in left-handed leptons, that experiences direct washout). In this work, the effect is quantified and explained in some detail on numerical examples for Leptogenesis.

Interestingly, the intermediate enhancement can be present in either case, whether the partially equilibrated spectator processes are leptonic or non-leptonic.

The outline of this paper is as follows: In Section 2, we review the well-known Boltzmann equations for Leptogenesis in the presence of spectator effects, mainly for the purpose of introducing our notations and conventions. The various temperature scales where the equilibration of particular SM processes occur are discussed in Section 3. As for the Yukawa-mediated processes, we make use of perturbative techniques and results that were recently presented in Refs. [8–10], while the rates of the instanton-mediated processes are taken or deduced from lattice simulations reported in Refs. [11, 12]. The results derived in Refs. [8–10] for leptonic processes are generalised to interactions involving quarks in Appendix A. As an example of the effect that is the main topic of this work, we calculate the impact of partial equilibration of τ -lepton Yukawa couplings in Section 4. This is of interest, as one should expect here the largest effect from the partial equilibration on the freeze-out asymmetry, as part of the asymmetry (in principle up to one third) is hidden from washout in the right-handed τ -leptons. However, in the present work, we neglect effects from partial flavour decoherence, that should be included in future work along the lines of Ref. [7]. Instead in Section 5 we consider a high temperature example, the equilibration of strong and weak sphalerons and b -quark Yukawa interactions, in a temperature regime where lepton flavour effects are negligible. It turns out that even in the absence of flavour effects, parts of the asymmetry can be hidden from washout in the right-handed b -quarks and in lepton doublets that are not directly affected by washout. Finally, we summarise and discuss the results in Section 6.

2 Boltzmann Equations for Leptogenesis

In this Section, we review the standard Boltzmann equations for Leptogenesis that are valid when interactions with spectator fields are either negligible (interaction rates much smaller than the Hubble rate) or when these are fully equilibrated (rates much above the Hubble rate). One purpose of this discussion is to introduce notations and conventions used in the present paper. The equations presented here are generalised to account for partial equilibration of the spectator processes in the subsequent Sections.

The Boltzmann equations usually are derived by substituting cross sections from Quantum Field Theory (QFT) into the collision term. An important caveat in that approach is that a subtraction of real intermediate states is necessary in order to reproduce a unitary evolution of the system, such that no asymmetry is generated in equilibrium, as it is required by the CPT (charge-parity-time) theorem. Alternatively, the Boltzmann equations for Leptogenesis can be derived from the Schwinger-Dyson equations of Non-Equilibrium QFT [13–15], which are unitary by construction. The resulting equations from both methods agree in the limit of the strong washout regime, what is our main concern here. The methods of obtaining solutions for the freeze-out lepton asymmetries and the properties of these are discussed in detail in Ref. [16].

In order to account for the expansion of the Universe, it is useful to introduce the

conformal time η , that is related to the comoving time t as $dt = a d\eta$, where the scale factor in a radiation-dominated Universe is $a = a_R \eta$. We take

$$a_R = T_{\text{com}} = \frac{m_{\text{Pl}}}{2} \sqrt{\frac{45}{g_* \pi^3}}, \quad (1)$$

where $m_{\text{Pl}} = 1.22 \times 10^{19}$ GeV is the Planck mass, $g_* = 106.75$ is the number of relativistic degrees of freedom and T_{com} is the comoving temperature (in terms of which the physical temperature is $T = T_{\text{com}}/a$). By this choice, the comoving time is simply given by $\eta = 1/T$. Now, an asymmetry in left-handed SM leptons ℓ is generated from the decay of sterile right-handed neutrinos N_i . We take the masses of M_i to be hierarchical, *i.e.* $M_1 \ll M_2, M_3$. When considering the out-of-equilibrium dynamics of N_1 only, the time evolution can be conveniently parametrised through the dimensionless variable $z = M_1/T$, such that the Hubble expansion rate is given by $H(z) = a_R^{-1} M_1^2 / z^2$.

We denote asymmetries in a particle species X as $q_X = n_X - n_{\bar{X}}$, where the charge densities q_X and number densities n_X refer to a single gauge degree of freedom. For small deviations from chemical equilibrium, the charges are related to the chemical potentials via $q_X = T^2/6\mu_X$ for a Weyl fermion and $q_X = T^2/3\mu_X$ for a complex scalar. Finally it is useful to introduce entropy normalised quantities $Y_X = q_X/s$, where $s = g_*(2\pi^2/45)T^3$. Since the sterile neutrinos are Majorana particles, these must be described in terms of their number densities n_{N_i} instead of charge densities. We define n_{N_i} to include the internal degrees of freedom, *i.e.* as the sum over the two helicity components, and use the entropy normalised number densities $Y_{N_i} = n_{N_i}/s$.

When spectator and flavour effects are neglected, the Boltzmann equations for Leptogenesis are given by

$$\frac{d}{dz} Y_{N_1} = -\bar{\mathcal{C}}_{N_1} (Y_{N_1} - Y_{N_1}^{\text{eq}}), \quad (2a)$$

$$\frac{d}{dz} Y_\ell = \frac{1}{2} \bar{S} (Y_{N_1} - Y_{N_1}^{\text{eq}}) - \bar{W} \left(Y_\ell + \frac{1}{2} Y_H \right), \quad (2b)$$

where $\bar{\mathcal{C}}$ is the thermally averaged decay rate of N_1 , \bar{S} the source term and \bar{W} the washout term.

For the simplest scenarios of Leptogenesis and in the strong washout regime, the freeze-out asymmetry is independent of the evolution at early times, and thus approximations that are valid for $z > 1$ can be applied: One may treat N_1 in the non-relativistic limit and employ Maxwell distributions instead of Fermi-Dirac and Bose-Einstein distributions. This results in the following expressions for the various thermally averaged

rates:

$$\bar{\mathcal{C}}_{N_1} = |Y_1|^2 \frac{z T_{\text{com}}}{8\pi M_1}, \quad (3a)$$

$$\bar{S} = \varepsilon \bar{\mathcal{C}}_{N_1} \quad \text{with} \quad \varepsilon = \frac{\Gamma_{N_1 \rightarrow \ell H} - \Gamma_{N_1 \rightarrow \bar{\ell} H^\dagger}}{\Gamma_{N_1 \rightarrow \ell H} + \Gamma_{N_1 \rightarrow \bar{\ell} H^\dagger}}, \quad (3b)$$

$$\bar{W} = |Y_1|^2 \frac{3 T_{\text{com}} e^{-z} z^{\frac{5}{2}}}{2^{\frac{7}{2}} \pi^{\frac{5}{2}} M_1}, \quad (3c)$$

$$Y_{N_1}^{\text{eq}} = 2 \frac{T_{\text{com}}^3 z^2}{2\pi^2} K_2(z) \frac{1}{s} \approx 2^{-\frac{1}{2}} \pi^{-\frac{3}{2}} z^{\frac{3}{2}} e^{-z} \frac{T_{\text{com}}^3}{s}, \quad (3d)$$

where Y_1 is the Yukawa coupling of N_1 to ℓ and H , and $K_2(z)$ is a Bessel function. Note that in general, ℓ is a linear combination of the three SM lepton doublets. In Eq. (3b), we employ the usual definition of the decay asymmetry ε , where $\Gamma_{N_1 \rightarrow \ell H}$ ($\Gamma_{N_1 \rightarrow \bar{\ell} H^\dagger}$) denotes the partial decay rate of N_1 into leptons (antileptons). Note that $\bar{\mathcal{C}}_{N_1}$ is the thermal average of the total decay rate of N_1 , whereas Y_ℓ accounts for one component of the gauge multiplet only. This is the origin of the explicit factor $\frac{1}{2}$ in front of the source term in Eq. (2b).

A key quantity controlling the dynamics of Leptogenesis is

$$K_1 = \bar{\mathcal{C}}_{N_1}(z = 1). \quad (4)$$

The basic regimes for Leptogenesis are typically separated between strong and weak washout, defined by $K_1 > 1$ and $K_1 < 1$, respectively. In the strong washout regime, to a good approximation, the lepton asymmetry freezes out when $z \approx z_f$, where [16]

$$z_f \approx 1 + 0.5 \log(1 + 0.0031 K_1^2 \log^5(9.6 K_1^2)). \quad (5)$$

To guide the eye in our presentation of the numerical results in Figures 2 and 3 below, we quote the values $z_f(K_1 = \{1, 10, 10^2, 10^3, 10^4\}) = \{1.1, 5.2, 8.8, 12.0, 14.9\}$ and also refer to the graph of that function that is presented in Ref. [16].

Since in the strong washout regime and for thermal initial conditions of the sterile neutrinos, $(Y_{N_1} - Y_{N_1}^{\text{eq}}) \ll Y_{N_1}^{\text{eq}}$, the solution to Eq. (2b) is to a good approximation

$$Y_{N_1} - Y_{N_1}^{\text{eq}} = \frac{1}{\bar{\mathcal{C}}_{N_1}(z)} \frac{d}{dz} Y_{N_1}^{\text{eq}}(z), \quad (6)$$

which we employ for the numerical results presented in this paper. This relation also holds for non-thermal initial conditions provided $z \gg 1$. As we see below, the outcome of Leptogenesis with partially equilibrated spectators to some extent also depends on contributions from $z \lesssim 1$, which is why we restrict the analysis in this paper to the case of thermal initial conditions. Note that inaccuracies in the calculation of $\bar{\mathcal{C}}_{N_1}$ cancel when substituting the deviation from equilibrium (6) into the source term (3b). Remaining uncertainties concern the decay asymmetry ε , but as this quantity is seen to increase

towards small values of z in Refs. [13, 14], our approximations for the enhancement of the freeze-out asymmetry from partially equilibrated spectators should be conservative.

Now, the interactions that can change Y_ℓ and Y_H without changing the baryon-minus-lepton asymmetry Y_{B-L} are usually referred to as spectator effects. In order to take account of these, Eq. (2b) must be accordingly modified. The condition of a vanishing weak hypercharge in conjunction with equilibrium conditions can be used to determine the value of Y_H . Furthermore, lepton number is not conserved when the weak sphalerons come into thermal equilibrium, and one should instead use $\Delta = B - L$ as a conserved charge, which is only broken by interactions mediated by the N_i .

The usual treatment of spectator effects is to assume that a certain interaction is either fully equilibrated or not at all. Then it is straightforward to write down an evolution equation for $Y_{B-L} \equiv Y_\Delta = \Delta/s$ in the form [2]

$$\frac{d}{dz}Y_\Delta = -\bar{S}(Y_{N1} - Y_{N1}^{\text{eq}}) + 2\bar{W}\left(c_\ell + \frac{1}{2}c_H\right)Y_\Delta, \quad (7)$$

where the coefficients c_ℓ , c_H are defined via $Y_\ell = c_\ell Y_\Delta$, $Y_H = c_H Y_\Delta$ and can be determined using the equilibrium conditions (see Section 5 for more details). Note that Y_Δ is defined including the sum over gauge degrees of freedom. When at lower temperatures interactions mediated by lepton Yukawa couplings are in equilibrium, one must distinguish between the particular asymmetries $\Delta_i = B/3 - L_i$, where $i = e, \mu, \tau$ [3, 5, 6]. When arranging the Y_{Δ_i} in a column vector, c_ℓ and c_H can be written as a matrix and a row-vector in lepton flavour space, respectively. This is the usual approach pursued in calculations on flavoured Leptogenesis [3, 6]. In case of incomplete flavour equilibration, one should take account of the correlations between the different lepton flavours, as it is described in Refs. [7, 17]. Combining the partial flavour decoherence with partial equilibration of spectator effects will be the topic of future work. In the examples that we discuss in this paper, we neglect the flavour effects.

The approach of assuming either complete equilibration of spectators or no equilibration at all immediately leads to the question when precisely these limits are valid and it requires the knowledge of the rates for the spectator processes. An overview of these rates and references to their derivations is presented in the following Section.

3 Rates for Spectator Processes

In the following, we review the interactions that equilibrate at time scales relevant for Leptogenesis, i.e. for temperatures $T \lesssim 10^{14} \text{ GeV}$, and that can mediate spectator processes. In particular these are the strong and weak sphalerons and the Yukawa interactions of various quarks and leptons. Gauge and top Yukawa interactions equilibrate above these scales and will not be considered here.

The method of obtaining the sphaleron rate at high temperatures is developed in Ref. [11]. It is applied to the SM, in particular including the effects of a Higgs boson, in

Ref. [12], where it is found that the diffusion rate for the Chern-Simons number is

$$\Gamma_{\text{sph}} = (8.24 \pm 0.10) \left(\frac{N}{2} \right)^5 \frac{g^2 T^2}{m_D^2} \left(\log \frac{m_D}{g^2 T} + 3.041 \right) \alpha^5 T^4, \quad (8)$$

where g is the $\text{SU}(N)$ gauge coupling, $\alpha = g^2/(4\pi)$ and m_D the Debye mass of the $\text{SU}(N)$ gauge bosons. The numerical coefficients have strictly been derived for $N = 2$, while the explicit scaling $\sim N^5$ is conjectured in Ref. [18], *cf.* the comments below. In the SM at high temperatures, the Debye mass-square of the W bosons is $m_D^2 = \frac{11}{6} g_2^2 T^2$, and we then obtain the weak sphaleron rate $\Gamma_{\text{ws}} = \Gamma_{\text{sph}}|_{g=g_2}$, where we set $g_2 = 0.55$ as suggested by renormalisation group running for temperatures $\sim 10^{12}$ GeV. There is also recent work that reports the sphaleron rate in the SM during the crossover between the high-temperature regime and low temperatures, where the electroweak symmetry is broken [19]. The Chern-Simons number diffusion rate enters the kinetic equations in the form [19–21]

$$\frac{d(B/3 + L_i)}{dt} = -\frac{1}{T} \Gamma_{\text{ws}} \sum_{i=1}^{N_f} (3\mu_{Qii} + \mu_{Lii}), \quad (9)$$

where $N_f = 3$ is the number of flavours in the SM. See also [22] for a simple and pragmatic discussion on how to substitute Γ_{ws} into the kinetic equations. On the left hand side, we may as well replace the charge densities B and L_i with $6q_{Qii}$ and $2q_{Lii}$. As stated in Section 2 above, we use the convention that q_X denotes the charge density of a species X of a particular entry of a gauge multiplet. Furthermore, we allow in this Section the charge densities and chemical potentials to be matrices, as this allows to account for possible correlations in flavour space. This is done in view of future work where we aim to include decohering flavour correlations. In the remainder of this paper, we set the flavour correlations to be zero and denote the diagonal components of the flavour correlation matrices using a single index, *i.e.* $q_{Xi} = q_{Xii}$.

Using these definitions and relations, we can break Eq. (9) down to

$$\frac{dq_{Qij}}{dt} = -\frac{\Gamma_{\text{ws}}}{2T^3} \delta_{ij} (9\text{tr}[q_Q] + 3\text{tr}[q_L]), \quad (10a)$$

$$\frac{dq_{Lij}}{dt} = -\frac{\Gamma_{\text{ws}}}{2T^3} \delta_{ij} (9\text{tr}[q_Q] + 3\text{tr}[q_L]), \quad (10b)$$

where the traces are taken over flavour space. The non-zero eigenvalues associated with this homogeneous system of differential equations are $18\Gamma_{\text{ws}}/T^3$. Taking $g_2 \approx 0.55$ for the $\text{SU}(2)$ gauge coupling of the SM around 10^{12} GeV, we find that $18\Gamma_{\text{ws}}/T^3 = H$ for $T = T_{\text{ws}} \approx 1.8 \times 10^{12}$ GeV. In the absence of other interactions this would be the weak sphaleron relaxation temperature, but including top Yukawa and strong sphalerons as spectators reduces the relevant temperature by a factor of two [22].

For the strong $\text{SU}(3)$ interactions, the sphaleron rate is perhaps less well known than for the weak $\text{SU}(2)$ case. The question is addressed in Refs.[18, 23], where the more

recent of these suggests that the sphaleron rate in $SU(N)$ theory scales as $\sim N^5$. The Debye mass-square is now given by $m_D^2 = 2g_3^2 T^2$ and we take $g_3 = 0.61$ for $T \sim 10^{12}$ GeV. Now, the strong sphalerons contribute to the kinetic equations as

$$\frac{dq_{Qij}}{dt} = -\frac{dq_{uij}}{dt} = -\frac{dq_{dij}}{dt} = -\frac{1}{3} \frac{\Gamma_{ss}}{T^3} \delta_{ij} (6\text{tr}[q_Q] - 3\text{tr}[q_u] - 3\text{tr}[q_d]), \quad (11)$$

where $\Gamma_{ss} = \Gamma_{\text{sph}}|_{g=g_3}$. Here the non-zero eigenvalue associated with the system of equations is $12\Gamma_{ss}/T^3$. Equilibration should therefore occur when $12\Gamma_{ss}/T^3 = H$, which implies an equilibration temperature of $T_{ss} \approx 2.4 \times 10^{13}$ GeV. Again this is slightly reduced when the top Yukawa is taken into account.

The calculation of the Yukawa-mediated rates at high temperatures requires a proper account of $2 \leftrightarrow 2$ processes involving gauge radiation. In the hot plasma, these rates turn out to be dominated by the t -channel exchange of fermions. For the production of light singlet fermions, a calculation is reported in Refs. [8, 9]. Using CTP techniques, the results are confirmed in Ref. [10], where in addition, also the rates involving the leptonic SM Yukawa couplings are obtained. It is found that

$$\frac{dq_L}{dt} = -\frac{\gamma^{\text{fl}\delta\ell}}{2} (h^\dagger h q_L + q_L h^\dagger h) + \gamma^{\text{fl}\delta\ell} h^\dagger q_R h, \quad (12a)$$

$$\frac{dq_R}{dt} = -\gamma^{\text{fl}\delta\ell} (h h^\dagger q_R + q_R h h^\dagger) + 2\gamma^{\text{fl}\delta\ell} h q_L h^\dagger, \quad (12b)$$

where $\gamma^{\text{fl}\delta\ell} \approx 5 \times 10^{-3} T$ (see also Ref. [26] for an earlier estimate). The Yukawa matrix h is diagonal in the mass eigenbasis of the charged leptons. The equilibration temperature for the τ flavour, where $h_{\tau\tau}^2 \gamma^{\text{fl}\delta\ell} = H$ is then given by $T_\tau \approx 3.7 \times 10^{11}$ GeV. The corresponding temperature for μ leptons is given through $h_{\mu\mu}^2 \gamma^{\text{fl}\delta\ell} = H$ as $T_\mu \approx 1.3 \times 10^9$ GeV.

In turn, the rate for equilibration of the up and down quark Yukawa interactions has not yet been studied in detail. As it is explained in Appendix A, the intermediate results presented in Ref. [10] can however be employed to obtain this rate as $\gamma^{\text{fl}\delta d} \approx \gamma^{\text{fl}\delta u} \approx 1.0 \times 10^{-2} T$ (the superscripts d/u indicate the down/up-type quark), at temperatures $T \sim 10^{12}$ GeV. For b -quarks, this implies an equilibration temperature of $T_b = 4.2 \times 10^{12}$ GeV. While the running of couplings can be mostly neglected for the charged lepton Yukawa interactions, the effect is notable as $\gamma^{\text{fl}\delta d}$ increases to $1.2 \times 10^{-2} T$ at $T \sim 10^9$ GeV (strange quark Yukawa equilibration) and to $1.5 \times 10^{-2} T$ at $T \sim 10^6$ GeV (down/up equilibration).

A summary of the various equilibration temperatures T_X , where the interaction rates agree with the Hubble rate, is given in Table 1. It should be noted that the equilibration temperatures are affected by spectators (in a similar way as the washout), so the temperatures given in the table should only serve as guidelines. A pictorial overview is presented in Figure 1, where we show the bands ranging from T_X to $20 T_X$. The choice of the location and widths of the bands depends of course on the requirements on the precision of the Boltzmann equations. By comparing with the numerical examples in the subsequent Sections, one should be able to see this in more detail, and one may conclude that partial equilibration of spectator effects is not an exceptional situation.

T_{ss}	$2.4 \times 10^{13} \text{ GeV}$	strong sphaleron
T_{ws}	$1.8 \times 10^{12} \text{ GeV}$	weak sphaleron
T_b	$4.2 \times 10^{12} \text{ GeV}$	bottom-quark Yukawa
T_c	$3.8 \times 10^{11} \text{ GeV}$	charm-quark Yukawa
T_s	$2.5 \times 10^9 \text{ GeV}$	strange-quark Yukawa
T_u	$1.9 \times 10^6 \text{ GeV}$	up-quark Yukawa
T_d	$8.8 \times 10^6 \text{ GeV}$	down-quark Yukawa
T_τ	$3.7 \times 10^{11} \text{ GeV}$	τ -lepton Yukawa
T_μ	$1.3 \times 10^9 \text{ GeV}$	μ -lepton Yukawa
T_e	$3.1 \times 10^4 \text{ GeV}$	electron Yukawa

Table 1: Equilibration temperatures T_X for Yukawa- and instanton-mediated SM processes. Methods of the calculations and uncertainties are discussed in the text. Partial equilibration is relevant when the freeze-out of the lepton asymmetry happens at temperatures between T_X and $20 T_X$.

In the literature (see e.g. [2, 3]), it is often assumed that the strong sphalerons equilibrate before bottom and tau Yukawas, and that the weak sphalerons equilibrate before second generation Yukawas. Here instead we note that it seems more appropriate to assume that the weak sphaleron equilibrates after the bottom but before tau Yukawas, and that the strong sphaleron might not be fully equilibrated when the bottom Yukawa becomes relevant. Overall it seems that in most regions more than one spectator interaction can be partially equilibrated. Noting these partial discrepancies, one should however keep in mind systematic uncertainties, in particular in the determination of the strong sphaleron rate.

4 Partial τ Yukawa Equilibration During Leptogenesis

For Leptogenesis in the strong washout regime, a comparably large lepton asymmetry is present in the plasma around $z \sim 1$, after which it is suppressed from washout until the freezes out occurs at $z \sim 10 - 20$, *cf.* the dashed line in Figure 2 for example. Spectator effects can transfer parts of the asymmetry into particles that are not subject to washout, like right-handed charged leptons. If the spectator interaction is fully equilibrated, the asymmetry in spectator fields closely tracks the lepton doublet asymmetry, such that this effect is adequately described by a modified washout factor, as implemented in Eq. (7).

However this description is not valid if the spectator effects equilibrate during the same times the lepton asymmetry freezes out. In that case, parts of the large asymmetry around $z \sim 1$ can be transferred to the spectator fields, but they will only fully equilibrate

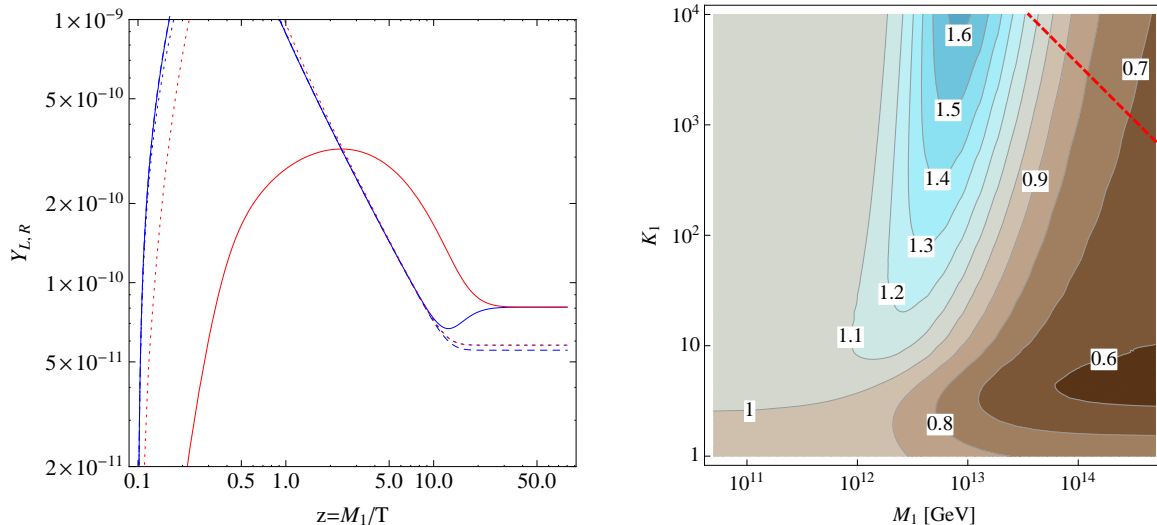


Figure 2: Left: Solution of the evolution equations (13) for $M_1 = 4 \times 10^{12}$ GeV and $Y_1 = 0.3$. Shown are solutions for $h_\tau = \sqrt{2}m_\tau/v$ (solid), for $h_\tau = 0$ (dashed) and for fully equilibrated tau Yukawa, $h_\tau \rightarrow \infty$, (dotted). Right: Enhancement of the asymmetry from partial spectator equilibration. Shown is the ratio of the freeze-out asymmetry $2Y_\ell + Y_R$ for the case $h_\tau = \sqrt{2}m_\tau/v$ divided by the case $h_\tau \rightarrow \infty$, as function of the lightest right-handed neutrino mass M_1 and of the washout strength K_1 as defined in Eq. (4). As expected, in the low and high M_1 limits this ratio goes to unity and $2/3$, respectively. Finally the region above the dashed red line would require non-perturbatively large Yukawa couplings ($Y_1^2 \geq 4\pi$) to realise the strong washout, and is therefore unphysical.

with the lepton doublet after the washout ends, thus *hiding* parts of the asymmetry from washout. As we show in the following, this effect can have a significant numerical impact on the predicted final asymmetry, compared to both the cases where spectator effects are neglected or assumed to be fully equilibrated.

For illustration, we consider here first a simplified scenario where the τ -Yukawa coupling equilibrates during Leptogenesis. A part of the lepton asymmetry is directly transferred to right-handed τ -leptons and hidden from washout processes. We expect the resulting effect to be larger than for non-leptonic spectator processes, where part of the hypercharge asymmetry that is initially present in Higgs bosons is distributed over a larger number of quark degrees of freedom. For simplicity, we ignore here other fully equilibrated spectator processes as well as flavour effects and the partial equilibration of weak sphalerons, that should occur at the same time. Therefore, the results do not serve the purpose of a full phenomenological study, but will rather indicate the size of the effect and for which range of parameters it may be important. A more realistic scenario is presented in the next Section.

To account for the partial equilibration of left handed and right-handed leptons ℓ and

R, the Boltzmann equations for the leptons (2b) or (7) are now replaced by

$$\frac{dY_\ell}{dz} = \frac{\bar{S}}{2} (Y_{N1} - Y_{N1}^{\text{eq}}) - \bar{W}Y_\ell - h_\tau^2 \gamma^{\text{fl}\delta\ell} \frac{T_{\text{com}}}{TM_1} (Y_\ell - Y_R), \quad (13a)$$

$$\frac{dY_R}{dz} = -2h_\tau^2 \gamma^{\text{fl}\delta\ell} \frac{T_{\text{com}}}{TM_1} (Y_R - Y_\ell), \quad (13b)$$

where $Y_{\ell,R} = q_{\ell,R}/s$ denote the asymmetries in a single gauge component, as before, and where $h_\tau \equiv h_{\tau\tau}$. Note that compared to Eqs. (12) a factor $T_{\text{com}}/(TM_1)$ arises from the change of the time variable from dt to dz . This also cancels the explicit T dependence in $\gamma^{\text{fl}\delta\ell}$. The evolution of N_1 is still determined by Eq. (2a) and the approximate solution (6).

Numerical solutions to Eqs. (13) are presented in Figure 2. In the left panel, we show the time evolution of the asymmetries $Y_{\ell,R}$ for one particular parameter point with the SM value of the lepton-Yukawa coupling

$$h_\tau = \sqrt{2}m_\tau/v, \quad (14)$$

where m_τ is the mass of the τ -lepton $v = 246$ GeV and $M_1 = 4 \times 10^{12}$ GeV and $Y_1 = 0.3$ (solid lines). For comparison, there are also the evolutions of the asymmetry in the absence of tau Yukawa interactions, $h_\tau = 0$, (dashed lines) and the evolutions in the fully equilibrated case, $h_\tau \rightarrow \infty$ (dotted lines). In all cases, the freeze out of the asymmetries is clearly visible around $z = 10$. For $h_\tau \neq 0$, we see that the right-handed asymmetry (red) is not equilibrated fast enough to follow the asymmetry in the lepton doublets (blue). It slowly increases until Y_ℓ drops below Y_R , and it is much larger than Y_ℓ at the time where the washout processes freeze out. The left-handed asymmetry is then replenished until $Y_\ell = Y_R$, and is about 40% larger than in the case where the tau Yukawa is assumed to be fully equilibrated (dotted lines), and where the right-handed asymmetry closely follows the one in the left-handed doublet.

In the right panel of Figure 2, we show the magnitude of this effect as a function of the Leptogenesis scale and of the washout strength K_1 as given in Eq. (4). The asymmetry $2Y_\ell + Y_R$ obtained from the solution to Eq. (13) for $h_\tau = \sqrt{2}m_\tau/v$ is divided by the asymmetry in the limit where the τ Yukawa interactions are fully equilibrated, $h_\tau \rightarrow \infty$. As expected, this ratio goes to unity for $M_1 \ll T_\tau$, while it goes to 2/3 for $M_1 \gg T_\tau$ where the asymmetry is only distributed among the two left-handed degrees of freedom. More importantly, we note that the asymmetry can be enhanced for intermediate values of M_1 compared to these limiting cases. While the enhancement is strongest in the regime where the washout is very large, it is still sizeable in regions with moderately strong washout of $K_1 \sim 10 - 100$. Furthermore it should be noted that the region where neither approximation gives an accurate result spans more than an order of magnitude, roughly from $3 \times T_\tau$ up to $60 \times T_\tau$, when demanding a 10% accuracy. Leptogenesis calculations that aim for a precision that is better than order one should therefore include these effects, where necessary.

5 Partial Equilibration of the b -Quark Yukawa Interactions during Leptogenesis

In the previous Section 4, we have treated a single lepton flavour during the equilibration of τ -Yukawa interactions only, which cannot be justified for realistic scenarios. A proper treatment would require to take account of lepton flavour decoherence along the lines of Refs. [7, 17], which is a considerable complication beyond the scope of the present paper. Therefore, we now consider Leptogenesis from the out-of-equilibrium decay of a singlet Majorana neutrino N_1 with mass of order 10^{13}GeV , which is much above T_τ and hence in the unflavoured regime, such that we can effectively consider the production and the washout of a lepton asymmetry within a single linear combination of e, μ, τ , that we refer to as ℓ_\parallel . The discussion and results of this Section should therefore be applicable to realistic scenarios. The regime of partial equilibration of the b -quark Yukawa interactions overlaps with those of the weak and the strong sphalerons, such that we need to consider these effects in conjunction.

However, it is of interest that an intermediate enhancement of the freeze-out asymmetry may also result from non-leptonic spectator processes, *i.e.* from b -quark Yukawa and strong sphaleron interactions only. We therefore first perform the numerical analysis for a vanishing rate of the weak sphalerons.

In order to account for the partial equilibration of the b -quark Yukawa interactions and the strong sphaleron processes, we need to identify linear combinations of charges that are only violated by these particular interactions. We notice that q_{Q1} , the charge density of a single component in the left-handed quark multiplet of the first generation is only altered by strong sphalerons, while $\Delta_{\text{down}} = q_b - q_{d1}$ is only altered by b -quark Yukawa interactions. Of course, we could have chosen to use q_{Q2} and q_{d2} instead of the first generation charge densities at this point, *i.e.* it is understood that $q_{Q2} = q_{Q1}$ and $q_{d2} = q_{d1}$. Moreover, as long as the second-generation quark Yukawa-couplings are out of equilibrium, the charge densities for the right-handed up- and down-type quarks of the first two generations must agree, *i.e.* $q_{d1,2} = q_{u1,2}$, as we assume in the following. We can therefore remove any explicit reference to q_{Q2} from the equations by replacing it with q_{Q1} and of q_{d2} , $q_{u1,2}$ by replacing these with q_{d1} . Of course, the corresponding substitutions can be made for the chemical potentials as well.

We account for the lepton number violating processes through the charge density $\Delta_\parallel = B/3 - 2q_{\ell_\parallel}$ (with the factor two being due to the fact that ℓ_\parallel accounts for one doublet component only), which is strictly conserved by weak sphalerons, and where B is the baryon number density. While these are out-of-equilibrium at the temperatures under consideration (and we could as well use ℓ_\parallel instead of Δ_\parallel), the choice of Δ_\parallel is useful in view of extending the present analysis to lower temperatures.

Defining $Y_{\Delta\parallel} = \Delta_{\parallel}/s$, we arrive at the following network of Boltzmann equations:

$$\frac{dY_{N1}}{dz} = -\bar{\mathcal{C}}_{N1}(Y_{N1} - Y_{N1}^{\text{eq}}), \quad (15a)$$

$$\frac{dY_{\Delta\parallel}}{dz} = -\bar{S}(Y_{N1} - Y_{N1}^{\text{eq}}) + 2\bar{W}\left(Y_{\ell\parallel} + \frac{1}{2}Y_H\right), \quad (15b)$$

$$\frac{dY_{\Delta\text{down}}}{dz} = -\bar{\Gamma}_b\left[Y_b - Y_{Q3} + \frac{1}{2}Y_H\right], \quad (15c)$$

$$\frac{dY_{Q1}}{dz} = -\frac{\Gamma_{\text{ss}}}{2T^3}\frac{T_{\text{com}}}{TM_1}[6Y_{Q3} - 3Y_t - 3Y_b + 24Y_{Q1}]. \quad (15d)$$

Here $\bar{\Gamma}_b = h_b^2\gamma^{\text{fl}\delta\text{d}}T_{\text{com}}/(TM_1)$, and factors of $T_{\text{com}}/(TM_1)$ again arise from the change of variables compared to (12,11). In addition, the evolution of the density of N_1 is given by Eq. (2a). In the strong washout regime and for $z \gg 1$, we may again use the approximate solution (3), with Y_1 being the Yukawa coupling between N_1 and ℓ_{\parallel} .

Now, we need to relate the quantities $Y_{\Delta\parallel}$, $Y_{\Delta\text{down}}$ and Y_{Q1} that appear on the left-hand side of the Boltzmann equations to $Y_{\ell\parallel}$, Y_{Q3} , Y_t , Y_b , and Y_H to $Y_{\Delta\parallel}$ on the right-hand side. The necessary constraints can be obtained following Ref. [5], by using the equilibrium conditions

$$\mu_{Q3} - \mu_{tR} + \mu_H = 0 \quad (\text{top-quark Yukawa interactions}), \quad (16a)$$

$$\mu_{Q3} + 2\mu_{Q1} + 2\mu_{tR} + 4\mu_{d1} - \mu_{bR} - 2\mu_{d1} - \mu_{\ell\parallel} + 2\mu_H = 0 \quad (\text{weak hypercharge neutrality}), \quad (16b)$$

$$2\mu_{Q3} + \mu_{tR} + \mu_{bR} + 4\mu_{Q1} + 4\mu_{d1} = 0 \quad (B \text{ conservation}), \quad (16c)$$

$$2\mu_{Q3} + \mu_{tR} + \mu_{bR} - 2\mu_{Q1} - 2\mu_{d1} = 0 \quad (\text{equal quark flavour asymmetries}), \quad (16d)$$

where we have dropped chemical potentials that are identically zero. Note that Eq. (16d) follows from the flavour-diagonal nature of strong sphaleron interactions. The last two equations immediately imply $\mu_{Q1} = -\mu_{d1}$, which has implicitly been used above to simplify the right-hand side of Eqn. (15d). For the remaining quantities, we find

$$(Y_{\ell\parallel}, Y_{Q3}, Y_t, Y_b, Y_H)^t = A(Y_{\Delta\parallel}, Y_{\Delta\text{down}}, Y_{Q1})^t, \quad (17)$$

where

$$A = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{1}{18} & -\frac{5}{9} & \frac{5}{9} \\ -\frac{1}{9} & \frac{1}{9} & -\frac{1}{9} \\ 0 & 1 & -1 \\ -\frac{1}{3} & \frac{4}{3} & -\frac{4}{3} \end{pmatrix}. \quad (18)$$

Note that this matrix acts on the space of charge densities defined by Eq. (17) and is therefore a different quantity from the matrix c_ℓ and the vector c_H , that occur for flavoured Leptogenesis and act on the space of SM lepton flavours [3, 5, 6].

As can be verified explicitly from the numerical results, it turns out that the strong sphalerons have only a small effect on the charge redistribution, what alleviates the systematic uncertainties from the strong sphaleron rate to some extent. It is therefore a good approximation to take out Eq. (15d) from the system of kinetic equations (15) and replace it by the equilibrium condition

$$2\mu_{Q3} - \mu_{tR} - \mu_{bR} + 4\mu_{Q1} - 4\mu_{d1} = 0 \quad (\text{strong sphalerons}). \quad (19)$$

In that case, the equilibrium conditions relate the charges that appear in the kinetic equations as

$$(Y_{\ell\parallel}, Y_{Q3}, Y_b, Y_H)^t = A(Y_{\Delta\parallel}, Y_{\Delta\text{down}})^t, \quad (20)$$

where

$$A = \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{23} & -\frac{10}{23} \\ \frac{1}{46} & \frac{18}{23} \\ -\frac{7}{23} & \frac{24}{23} \end{pmatrix}. \quad (21)$$

Finally in order to quantify the importance of the partial equilibration, we also need to consider the case where the bottom Yukawa interactions and strong sphalerons are fully equilibrated. As stated above, fast strong sphaleron processes impose the constraint (19), which replaces the differential equation (15d). For fully equilibrated bottom Yukawa interactions, there is the additional the constraint

$$\mu_{Q3} - \mu_{bR} - \mu_H = 0, \quad (22)$$

that can be used to replace the differential equation (15c). This implies that all asymmetries can be expressed in terms of $Y_{\Delta\parallel}$ only, and we can solve Eq. (15b) using the relation

$$(Y_{L\parallel}, Y_H)^t = \left(-\frac{1}{2}, -\frac{1}{5}\right)^t Y_{\Delta\parallel}. \quad (23)$$

Note that these factors are different from the coefficients used in Ref. [2], since there it is assumed that both the bottom and tau Yukawa interactions equilibrate at the same time.

In Figure 3, we show the ratio of the asymmetry computed when accounting for the partial equilibration of the b -quark Yukawa interactions and strong sphalerons over the asymmetry that results when assuming a full equilibration. In order to illustrate the relatively small effect of the strong sphalerons, we include also the same plot obtained when taking the strong sphaleron interactions to be in equilibrium.

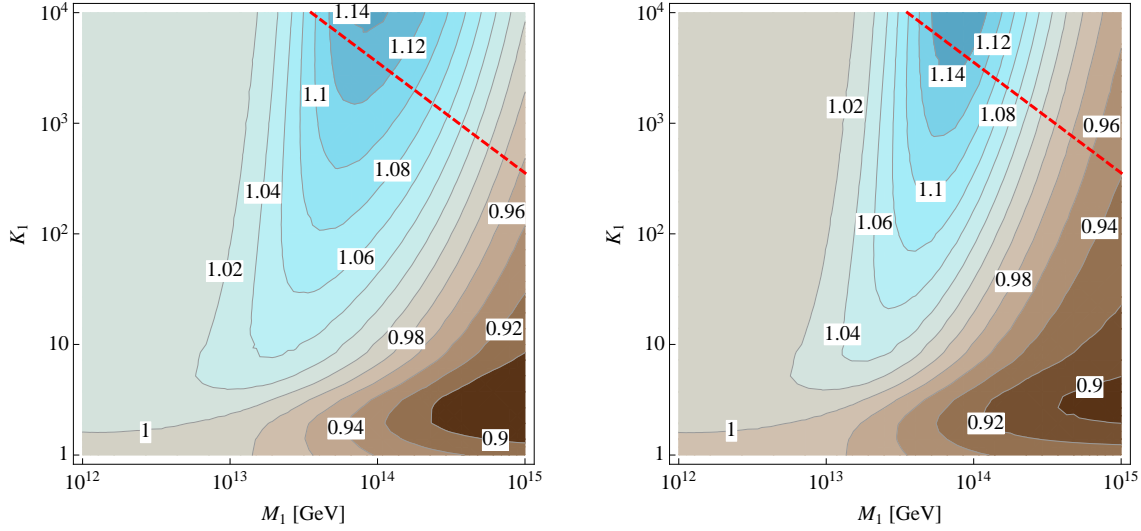


Figure 3: Left: Enhancement of the asymmetry from partial equilibration of b -quark Yukawa interactions and strong sphalerons as function of the lightest right-handed neutrino mass M_1 and of the washout strength K_1 as defined in the text. Right: Same as left, but with the strong sphaleron imposed to be in equilibrium. As before, in the regions above the dashed red lines Y_1 is non-perturbatively large.

Overall, the effect of partial bottom Yukawa equilibration is smaller than in the case of partial τ equilibration, exceeding the 10% level only in small corners of parameter space. The reason for this is that in the limit where the bottom Yukawa is in equilibrium, the constraint becomes $Y_{\Delta\text{down}} = Y_{\Delta\parallel}/10$, such that only a small fraction of the asymmetry is ever transferred into the quark flavour asymmetry Δ_{down} . In comparison, the equilibrium condition induced by the τ Yukawa corresponds to $Y_\ell = Y_R$, such that a larger fraction can be transferred and hidden.

Now, we turn to the realistic situation where weak sphalerons are active. Since we have convinced ourselves about the small effect from strong sphalerons, we assume for simplicity that these are fully equilibrated and account for the partial equilibration of b -quark Yukawa interactions and of weak sphalerons.

Once the weak sphalerons become active, baryon number is not conserved anymore, such that the constraint (16c) disappears and correspondingly a new variable appears in the Boltzmann equations. It is convenient to choose $q_{\ell\perp}$, where $\ell\perp$ can be either of the two linear combination of SM lepton flavours that is perpendicular to $\ell\parallel$, as variable, since in the absence of charged lepton flavour effects this charge is only violated by weak sphalerons. The corresponding Boltzmann equation that describes the weak sphaleron is given by

$$\frac{dY_{\ell\perp}}{dz} = -\frac{\Gamma_{\text{ws}}}{2T^3} \frac{T_{\text{com}}}{TM_1} (9Y_{Q3} + 18Y_{Q1} + 3Y_{\ell\parallel} + 6Y_{\ell\perp}), \quad (24)$$

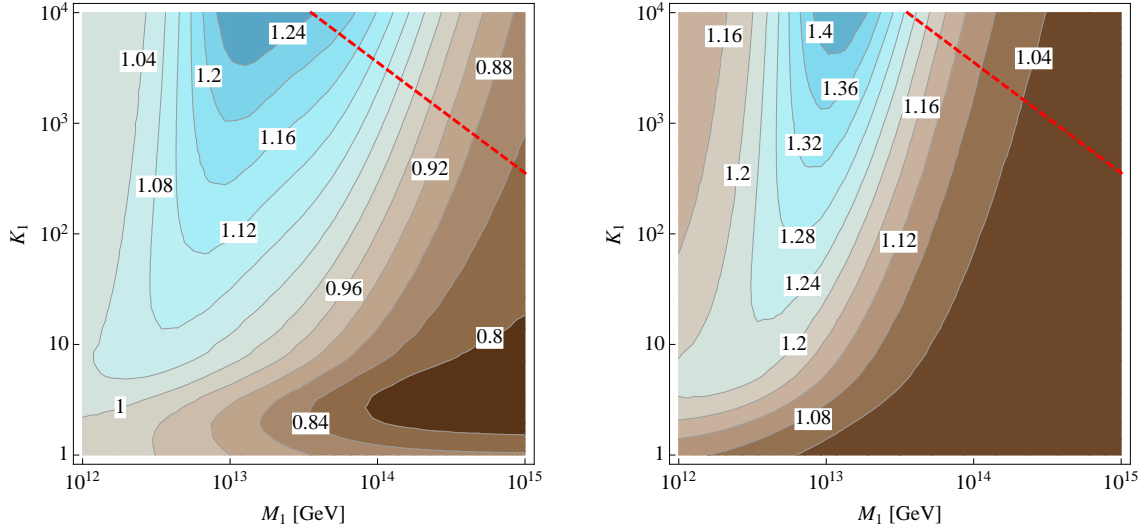


Figure 4: Left: Ratio of the asymmetry when taking account of partial equilibration of b -quark Yukawa interactions and weak sphalerons and the fully equilibrated approximation, as function of the lightest right-handed neutrino mass M_1 and of the washout strength K_1 as defined in the text. Right: Ratio of the result obtained with partially equilibrated b -quarks and weak sphalerons divided by the result obtained with partially equilibrated b -quarks but neglecting weak sphaleron interactions.

in addition to the equations for $Y_{\Delta\parallel}$ and $Y_{\Delta\text{down}}$. With the strong sphalerons in equilibrium, the constraint (19) is imposed on the chemical potentials. Solving the new system of equations we can again express the fields appearing on the right-hand sides of the Boltzmann equations using the relations

$$(Y_{\ell\parallel}, Y_{\ell\perp}, Y_{Q3}, Y_b, Y_{Q1}, Y_H) = A(Y_{\Delta\parallel}, Y_{\ell\perp}, Y_{\Delta\text{down}})^t, \quad (25)$$

where now

$$A = \begin{pmatrix} -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{23} & \frac{1}{2} & -\frac{10}{23} \\ \frac{1}{46} & \frac{1}{2} & \frac{18}{23} \\ -\frac{1}{46} & \frac{1}{2} & \frac{5}{23} \\ -\frac{7}{23} & 0 & \frac{24}{23} \end{pmatrix}, \quad (26)$$

and where $Y_{\ell\perp} = q_{\ell\perp}/s$ is the charge in a single gauge degree of freedom, as usual. The results for partially equilibrated b -quark Yukawa interactions and weak sphalerons are presented in Figure 4. We observe that the partial equilibration of weak sphaleron processes, that include leptons, have a stronger enhancing effect than the non-leptonic

processes considered above. Of course, this is because the weak sphalerons are more efficient in hiding asymmetries due to their direct coupling to the initial asymmetry within ℓ_{\parallel} . When comparing the left panel of Figure 4 with either of the plots in Figure 3 we observe that there is no clear separation of the regions where the enhancement can be attributed to b -quark interactions from regions where it results from weak sphalerons. Accounting for both interactions together apparently leads to a broadening of the region of enhancement. The purpose of the plot in the right panel of Figure 4 is to isolate the effect from the weak sphalerons only, by dividing out the b -quark effects. Finally, we mention that for values at the lower end of the range of M_1 that is presented in Figure 4, one should already expect effects from the τ -lepton Yukawa coupling to be of importance. As this however also requires to include flavour decoherence, the total effect will be more model-dependent, what should be subject of a future study.

6 Discussion and Conclusions

The following conclusions can be drawn from the numerical examples presented in Sections 4 and 5:

- As one should expect, the equilibration temperatures (*cf.* Figure 1 and Table 1), defined by the time when the respective reaction rates agree with the Hubble rate, indicate the location of the parametric regime where the partial equilibration of spectator effects is of importance. More precisely, in terms of the mass M_1 of the decaying right-handed neutrino, it ranges from roughly a factor of 3 above the equilibration temperature and spans two orders of magnitude, when aiming to calculate the spectator effect with a (relative) accuracy at the 10% level. In terms of M_1 , the partial equilibration regime is located above the equilibration temperature, because the freeze-out occurs at temperatures below M_1 , *cf.* Eq. (5). This is also a simple explanation of the shift of the partial equilibration regime toward larger values of M_1 in the case of stronger washout, as z_f increases slowly with K_1 .
- For the two examples (τ -Yukawa equilibration and b -Yukawa together with strong-sphaleron equilibration) worked out numerically in Sections 4 and 5, the freeze-out asymmetry assuming full equilibration of the spectator fields is larger than when these effects are neglected, in consistency with the well-known results presented in Refs. [2, 3]. For partial equilibration though, we find that the freeze-out asymmetry can be enhanced compared to the two limiting cases, *cf.* Figures 2-4. This observation is best explained by the graphs in the left panel of Figure 2. The enhancement is typically more pronounced the stronger the washout strength is. This is because during the early stages of Leptogenesis, the large asymmetries that are present can be partly transferred to spectator fields. When the spectators do not reach full chemical equilibrium with the fields ℓ or H prior to freeze out, asymmetries can be partly hidden from washout. Only after freeze out, chemical equilibration is

reached such that these asymmetries can be converted to a baryon asymmetry by weak sphaleron processes. This “hiding mechanism” is different from and more efficient than the case of full equilibration, where the asymmetries in the spectator fields track the evolution of the left-handed lepton asymmetries, what typically only results in a moderate reduction of the washout strength.

On possible future directions of research and applications of the present methods, we make the following remarks:

- In order to properly study the regime of partial equilibration of τ - and μ -Yukawa couplings, effects of partial flavour decoherence should be considered. A possible direction would be to combine the methods presented here with the treatment of flavour effects in Ref. [7].
- An enhancement of the asymmetries compared to the limiting cases of complete equilibration or complete absence of spectator effects is most pronounced for large washout strengths K_1 . Consequently, this requires large values of the Yukawa coupling Y_1 of the right-handed neutrinos. From the standard parametrisation of these couplings in the type-I see-saw mechanism [24], it can be seen that this is generally possible, but also that this requires cancellations in order to suppress the masses of the SM neutrinos. Scenarios with very strong washout can however be motivated and realised in the context of resonant Leptogenesis by appealing to approximate symmetries [25], such that the effects discussed in the present paper may be particularly sizeable.
- Instead of the renormalisable or non-perturbative rates that scale $\propto T$, it may be of interest to consider non-renormalisable interactions with rates $\propto T^n$, where $n > 1$, that mediate the initial asymmetry to the spectator sector. Such processes are faster at early times when large asymmetries are present, such that these can be more efficiently hidden from strong washout. This should allow for Baryo- and Leptogenesis from the out-of-equilibrium decay of particles that are more strongly coupled than the right-handed neutrinos in the standard scenarios of Leptogenesis, what may possibly lead to interesting phenomenological prospects. We note in that context that the effect of protecting primordial asymmetries from washout at later stages through what one may essentially call a non-renormalisable spectator effect was first pointed out in Ref. [26].
- The relations between the charge densities and chemical potentials receive thermal radiative corrections, that would enter the present calculations as a next-to leading-order effect, and that have been reported recently in Ref. [27]. Due to the infrared population of bosonic fields, these corrections turn out most sizeable for the Higgs boson. Future calculations with the goal of a high precision should also include these effects.
- It will be of interest to investigate the dependence of the freeze-out asymmetry on the initial conditions. An important feature of Leptogenesis in the strong washout

regime is that the freeze-out asymmetry is approximately independent on the initial distribution of the sterile neutrinos. In the present work, we always assume a thermal initial distribution for these particles. However, from Figure 2, we can see that due to partially equilibrated spectator particles, the outcome of Leptogenesis may be sensitive to the initial conditions after all. A substantial part of the asymmetry is produced and partly transferred to the spectators at small z . When assuming non-thermal (*e.g.* vanishing) initial conditions for the sterile neutrinos, the initial deviation from equilibrium $Y_{N1} - Y_{N1}^{\text{eq}}$ will be enhanced compared to the thermal case, and consequently a larger asymmetry within the spectator fields will be generated at these times. We therefore expect that in such a case the corrections to the standard calculations from partial equilibration of spectators will be more important than for the thermal initial conditions considered in this paper.

- In connection with the previous point, it will be important to improve the predictions quantitatively by treating the relativistic regime $z \ll M_1$ more accurately. For thermal initial distributions of the sterile neutrinos, we see from Figure 2 that about half of the initial spectator asymmetry is generated for $z \lesssim 0.5$, where the non-relativistic approximation for the rate of sterile neutrino interactions become in principle inapplicable. However, combining Eqs. (3b) and (6), we see that corrections to the rate $\bar{C}_{N1}(z)$ approximately cancel in the source term. Washout becomes quantitatively relevant only for larger z , where the non-relativistic approximations used here are valid. We therefore expect the present results not to change substantially upon improving on the calculation of $\bar{C}_{N1}(z)$ in the relativistic regime, $z \ll 1$. This may not be the case however when assuming vanishing initial conditions for the sterile neutrinos, where the deviation from equilibrium for $z \ll 1$ is large. The pertinent CP conserving rates are calculated in Refs. [8–10], and it should be straightforward to substitute these into the expressions for the CP -violating source in Ref. [14] in the hierarchical limit. When including spectator effects, it may therefore turn out that quantum statistical corrections and relativistic rates are of importance for Leptogenesis in the strong washout regime, in contrast to the most simple scenarios. We will investigate this possibility in future work.

Whether spectator effects and their partial equilibration should be included in a particular calculation of the lepton asymmetry is a matter of the precision that it aims for. Since the prospects for a precise measurement of the high scale parameters relevant for Leptogenesis are rather bad, one may perhaps decide to neglect this effect in a pragmatic phenomenological study. However once additional constraints on the parameter space are imposed, *e.g.* when one demands the washout of primordial asymmetries as in Ref. [28] or imposes the gravitino bound on the reheating temperature [29, 30], including these effects might be warranted or even necessary to gain a proper understanding of the viable parameter space. In addition, the insights gained may be helpful in identifying alternative mechanisms of Baryogenesis, where of particular interest are scenarios that are accessible to observational tests.

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A Rate of Quark-Yukawa Mediated Processes

By appropriate replacements of coupling constants and group-theoretical factors, the intermediate numerical results that are presented in Ref. [10] can be used in order to derive the coefficient $\gamma^{\text{fl}\delta d}$ for the equilibration rate of down-type Yukawa mediated processes. It can be decomposed as

$$\gamma^{\text{fl}\delta d} = \gamma^{\text{fl}(\phi)\delta d} + \gamma^{\text{fl}(\phi)\delta d} + \gamma^{\text{fl}(Q)\delta d} + \gamma^{\text{fl}(d)\delta d} + \gamma_{\text{vertex}}^{\text{fl}\delta d} + \gamma_{1\leftrightarrow 2}^{\text{fl}\delta d}, \quad (27)$$

where the individual terms are given by

$$\gamma^{\text{fl}(\phi)\delta d} = 7.71 \times 10^{-4} G^{(\phi)} T + 1.32 \times 10^{-3} h_t^2 T, \quad (28a)$$

$$\gamma^{\text{fl}(Q,d)\delta d} = 3.72 \times 10^{-3} G^{(Q,d)} T - 8.31 \times 10^{-4} G^{(Q,d)} T \log G^{(Q,d)}, \quad (28b)$$

$$\gamma_{\text{vertex}}^{\text{fl}\delta d} = -7.72 \times 10^{-4} \left(\frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{5}{18} g_1^2 \right) T, \quad (28c)$$

$$\gamma_{1\leftrightarrow 2}^{\text{fl}\delta d} = 1.7 \times 10^{-3} \left(\frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 \right) T, \quad (28d)$$

with

$$G^{(\phi)} = \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2, \quad (29a)$$

$$G^{(Q)} = \frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{18} g_1^2, \quad (29b)$$

$$G^{(d)} = \frac{8}{3} g_3^2 + \frac{2}{9} g_1^2. \quad (29c)$$

While the results (28a, 28b, 28c) are accurate to leading order in the couplings following the calculation of Ref. [10], the contribution (28d) corresponds to an estimate. A full calculation along the lines of Refs. [8, 31] may turn out to be somewhat involved, but we expect that Eqs. (28) should be suitable for the present purposes, in particular because the dominating exchange of quarks in the t -channel is accurately captured through Eq. (28b) to leading order. With the values for g_2 and g_3 that are stated in Section 3 and $g_1 = 0.41$ for temperatures of order 10^{12} GeV, Eq. (27) yields $\gamma^{\text{fl}\delta d} \approx 0.01$.

For completeness, we also estimate the rate for up-type Yukawa mediated interactions. Here the term proportional to h_t^2 in (28a) does not contribute, since the Yukawa coupling

alone does not break left-handed quark number (it only mediates processes like $Q\bar{Q} \leftrightarrow t_R\bar{t}_R$ or $Q\bar{t}_R \leftrightarrow Q\bar{t}_R$, but not e.g. $Q\bar{t}_R \leftrightarrow \bar{Q}t_R$). The vertex contribution becomes

$$\gamma_{\text{vertex}}^{\text{fl}\delta u} = -7.72 \times 10^{-4} \left(\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{17}{18}g_1^2 \right) T, \quad (30)$$

and $G^{(d)}$ should be replaced with

$$G^{(u)} = \frac{8}{3}g_3^2 + \frac{8}{9}g_1^2. \quad (31)$$

Altogether we obtain $\gamma^{\text{fl}\delta u} \approx 0.01 T$. The rates for up and down type quarks differ in the next digit, but this is beyond the level of accuracy of this estimate. For the charged lepton Yukawa interaction rates, the running of g_2 and g_1 induces a temperature dependence which is negligible for all practical purposes. On the other hand the QCD coupling runs more strongly, and the rates $\gamma^{\text{fl}\delta u} \approx \gamma^{\text{fl}\delta d}$ increase to $0.012 T$ at $T \approx 10^9$ GeV and to $0.015 T$ at $T \approx 10^6$ GeV. This effect is included in our estimates of the quark Yukawa equilibration temperatures in Tab. 1.

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